

Fill in the Table and look for any patterns

| | Quotient of Powers | Expanded Form | Simplify: Write answer using a positive exponent | Rewrite with a negative exponent |
|--------------|-------------------------|---------------------------------------|---|---|
| a) | $\frac{5^2}{5^3}$ | $\frac{5 \cdot 5}{5 \cdot 5 \cdot 5}$ | $\frac{1}{5}$ | $(5)^{-1}$ |
| b) | $\frac{5^3}{5^2}$ | $\frac{5 \cdot 5 \cdot 5}{5 \cdot 5}$ | $\frac{5}{1} = 5$ | $\frac{1}{5^{-1}}$ or $\left(\frac{1}{5}\right)^{-1}$ |
| c) | $\frac{x^5}{x^2}$ | | | |
| d) | $\frac{x^2}{x^5}$ | | | |
| e) | $\frac{x^{23}}{x^{14}}$ | | | |
| f) | $\frac{x^{14}}{x^{23}}$ | | | |
| g) | $\frac{x^{94}}{x^{36}}$ | | | |
| Rule: | | | | |

Quotient of Powers

Standards:

2.0 Understand and use the rules of exponents

10.0 Divide monomials. Solve multi-step problems, including word problems, by using this technique(Develop)

Objective: Today I will divide powers with the same base and raise a quotient to a power.

Lesson Opener:

Activity can be done as a class or students can fill in the chart in partners/groups.

Debrief:

“Did you notice any patterns?” [Answers will vary]

- same bases
- all had same coefficient
- Do not need to expand each expression. When you divide powers with the same base, subtract their exponents
- Negative exponent is the reciprocal of the base

to write using positive exponents

- When there are more factors in the numerator than denominator: x^n n is positive
- When there are more factors in the denominator than numerator: $\frac{1}{x^n}$

Quotient of Powers Property: Dividing powers with the same base

For every non-zero number a and integers m and n

$$\frac{a^m}{a^n} = a^{m-n} \quad \text{where } a \neq 0$$

Ex 1) Simplify each expression.(write using positive exponents)

a) $\frac{4^4}{4}$ $\frac{4^4}{4}$

$$= \frac{4 \cdot 4 \cdot 4 \cdot 4}{4} = 4^{4-1}$$

$$= 4^3 = 4^3$$

b) $\frac{2^5}{2^6}$ $\frac{2^5}{2^6}$

$$= \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2^{5-6}$$

$$= \frac{1}{2} = \frac{1}{2}$$

c) $\frac{z^7}{z^3}$ $\frac{z^7}{z^3}$

$$= \frac{z \cdot z \cdot z \cdot z \cdot z \cdot z \cdot z}{z \cdot z \cdot z} = z^{7-3}$$

$$= z^4 = z^4$$

d) $\frac{x^4 y^2}{x^2 y^3}$ $\frac{x^4 y^2}{x^2 y^3}$

$$= \frac{x \cdot x \cdot x \cdot x \cdot y \cdot y}{x \cdot x \cdot y \cdot y \cdot y} = x^{4-2} y^{2-3}$$

$$= \frac{x^2}{y} = \frac{x^2}{y}$$

Quotient of Powers

You try!
Simplify

$$\begin{aligned} \text{a) } \frac{x^3}{x^5} &= \frac{x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} \\ &= \frac{1}{x^2} \end{aligned} \quad \begin{aligned} \frac{x^3}{x^5} &= x^{3-5} \\ &= x^{-2} \\ &= \frac{1}{x^2} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{3x^6}{x^3} &= \frac{3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} \\ &= 3x^3 \end{aligned} \quad \begin{aligned} \frac{3x^6}{x^3} &= 3x^{6-3} \\ &= 3x^3 \end{aligned}$$

Ex 2) Simplify

$$\begin{aligned} \frac{3x^5 \cdot x^2}{6x^3} &= \frac{3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{2 \cdot 3 \cdot x \cdot x \cdot x} \\ &= \frac{x^4}{2} \end{aligned} \quad \begin{aligned} \frac{3x^5 \cdot x^2}{6x^3} &= \frac{3x^{5+2}}{6x^3} \\ &= \frac{3x^7}{2 \cdot 3 \cdot x^3} \\ &= \frac{x^{7-3}}{2} \\ &= \frac{x^4}{2} \end{aligned}$$

You Try!
Simplify

$$\begin{aligned} \text{a) } \frac{z^4 \cdot 10z^3}{z^2} &= \frac{10z^{4+3}}{z^2} \\ &= 10z^{7-2} \\ &= 10z^5 \end{aligned} \quad \begin{aligned} \frac{z^4 \cdot 10z^3}{z^2} &= z^4 \cdot 10z^{3-2} \\ &= z^4 \cdot 10z^1 \\ &= 10z^{4+1} \\ &= 10z^5 \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{x \cdot 7x^5}{9x^4} &= \frac{7x^{5+1}}{9x^4} \\ &= \frac{7x^6}{9x^4} \\ &= \frac{7x^{6-4}}{9} \\ &= \frac{7x^2}{9} \end{aligned} \quad \begin{aligned} \frac{x \cdot 7x^5}{9x^4} &= \frac{x \cdot 7x^{5-4}}{9} \\ &= \frac{x \cdot 7x}{9} \\ &= \frac{7x^2}{9} \end{aligned}$$

Quotient of Powers

Ex 3) Simplify by using repeated multiplication

$$\begin{aligned}
 \text{a)} \quad & \left(\frac{x}{y}\right)^2 \\
 &= \frac{x}{y} \cdot \frac{x}{y} \\
 &= \frac{x \cdot x}{y \cdot y} \\
 &= \frac{x^2}{y^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & \left(\frac{a}{b}\right)^3 \\
 &= \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b} \\
 &= \frac{a \cdot a \cdot a}{b \cdot b \cdot b} \\
 &= \frac{a^3}{b^3}
 \end{aligned}$$

“Do you see a pattern? Another way of simplifying without using repeated addition?”
 [Answers will vary]

Raising a quotient to a power property:
 For every non-zero number a and b and integer n ,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \text{ where } b \neq 0$$

Note: Distribution is over addition/subtraction only. The exponent is **NOT** being distributed

You try!
 Simplify

$$\begin{aligned}
 \text{a)} \quad & \left(\frac{-3}{y}\right)^3 \\
 &= \frac{-3}{y} \cdot \frac{-3}{y} \cdot \frac{-3}{y} \\
 &= \frac{-3(-3)(-3)}{y^3} \\
 &= \frac{-27}{y^3}
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{-3}{y}\right)^3 \\
 &= \frac{(-3)^3}{y^3} \\
 &= \frac{-3(-3)(-3)}{y^3} \\
 &= \frac{-27}{y^3}
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & \left(\frac{2m}{n}\right)^4 \\
 &= \frac{2m}{n} \cdot \frac{2m}{n} \cdot \frac{2m}{n} \cdot \frac{2m}{n} \\
 &= \frac{2(2)(2)(2)m^4}{n^4} \\
 &= \frac{16m^4}{n^4}
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{2m}{n}\right)^4 \\
 &= \frac{2^4 m^4}{n^4} \\
 &= \frac{2(2)(2)(2)m^4}{n^4} \\
 &= \frac{16m^4}{n^4}
 \end{aligned}$$

Quotient of Powers

Simplify. Write your answer using only positive exponents.

Ex 4) $\left(\frac{2m}{n}\right)^{-4}$

*Remind students that a negative exponent is the reciprocal of the base

$$\begin{aligned} & \left(\frac{2m}{n}\right)^{-4} \\ &= \left(\frac{n}{2m}\right)^4 \\ &= \frac{n}{2m} \cdot \frac{n}{2m} \cdot \frac{n}{2m} \cdot \frac{n}{2m} \\ &= \frac{n^4}{2(2)(2)(2)m^4} \\ &= \frac{n^4}{16m^4} \end{aligned}$$

$$\begin{aligned} & \left(\frac{2m}{n}\right)^{-4} \\ &= \left(\frac{n}{2m}\right)^4 \\ &= \frac{n^4}{2^4 m^4} \\ &= \frac{n^4}{2(2)(2)(2)m^4} \\ &= \frac{n^4}{16m^4} \end{aligned}$$